AUTOMATIC NORM CONTINUITY OF WEAK* HOMEOMORPHISMS

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Abstract. We prove that in a certain class $\mathcal{E}$ of nonseparable Banach spaces the norm topology of the dual ball is definable in terms of its weak* topology. Thus, if $X, Y \in \mathcal{E}$ and $f : B_{X^*} \longrightarrow B_{Y^*}$ is a weak*-to-weak* homeomorphism, then f is automatically norm-to-norm continuous.

1. Introduction

The aim of this note is to prove the automatic continuity in the norm topology for the weak* homeomorphisms of the dual ball of certain nonseparable Banach spaces. We start by observing that such a property never holds in the separable case.

Proposition 1. *Let X be a separable infinite dimensional Banach space. Then, there exists a weak*-to-weak* homeomorphism $f : B_{X^*} \longrightarrow B_{X^*}$ which is not norm-to-norm continuous.*

Proof: The space (B_{X^*}, w^*) is a metrizable infinite-dimensional compact convex set, so it is homeomorphic to the Hilbert cube $[0,1]^\omega$, by Keller's Theorem. It is a known fact, cf. [4, p. 261], that the Hilbert cube is countable dense homogenous, which means that if A and B are countable weak* dense subsets of B_{X^*} then there is a weak* homeomorphism $f : B_{X^*} \longrightarrow B_{X^*}$ such that $f(A) = B$. Thus, it is enough to find two such subsets A and B with some different property relative to the norm topology. For example, A can be taken so that its norm-closure is connected (by choosing it to be rationally convex) and B with disconnected norm-closure (take B' a countable weak* dense set which is not norm dense and then add to B' a point out of its norm-closure). $\square$

We shall introduce two classes $\mathcal{E}$ and $\mathcal{E}_0$ of nonseparable Banach spaces with the properties indicated in the two following theorems.

Theorem 2. *Let X and Y be spaces in the class $\mathcal{E}$ and let $f : B_{X^*} \longrightarrow B_{Y^*}$ be a weak*-to-weak* homeomorphism. Then f is norm-to-norm continuous.*

2000 *Mathematics Subject Classification.* 46B26.

The author was supported by a Marie Curie Intra-European Felloship MCEIF-CT2006-038768 (E.U.) and research projects MTM2005-08379 and Séneca 00690/PI/04 (Spain).

Theorem 3. *Let X and Y be spaces in the class $\mathcal{E}_0$ and let $f : B_{X^*} \longrightarrow B_{Y^*}$ be a weak*-to-weak* homeomorphism. Then, if (x_n^*) is a sequence in B_{X^*} which weak* converges to x^* and $\|x_n^*\| \longrightarrow \|x^*\|$, then $\|f(x_n^*)\| \longrightarrow \|f(x^*)\|$.*

The definition of these classes requires some preliminary work and will be given later but we can already indicate some examples of spaces which we know that belong in there. The most significant representatives in $\mathcal{E} \cap \mathcal{E}_0$ are the spaces $c_0(\Gamma)$ and $\ell_p(\Gamma)$ for $1 < p < \infty$, Γ being an uncountable set of indices. More generally, any uncountable c_0-sum or ℓ_p-sum ($1 < p < \infty$) of separable spaces belongs to $\mathcal{E}_0$, and any space from $\mathcal{E}_0$ with the dual Kadec-Klee property belongs to $\mathcal{E}$. Recall that X has the dual Kadec-Klee property if whenever we have a weak* convergent sequence (x_n^*) in the dual such that the sequence of norms $(\|x_n^*\|)$ converges to the norm of the limit, then actually the sequence is norm-convergent. In addition, the class $\mathcal{E}$ is closed under finite ℓ_1-sums. On the other hand, one can show that $\ell_1(\Gamma) \notin \mathcal{E} \cup \mathcal{E}_0$, indeed:

Proposition 4. *For any infinite set Γ, there is a weak*-to-weak* homeomorphism $f : B_{\ell_1(\Gamma)^*} \longrightarrow B_{\ell_1(\Gamma)^*}$ which is not norm-to-norm continuous and does not preserve the limit of norms of weak* convergent sequences.*

Proof: We know that $\ell_1(\Gamma)^* = \ell_\infty(\Gamma)$ and hence $B_{\ell_1(\Gamma)^*} \approx [-1,1]^\Gamma$ where the weak* topology is identified with the pointwise topology. Consider elements $e_1 = (1,0,0,\ldots)$, $e_2 = (0,1,0,0,\ldots)$, etc. in $[-1,1]^\Gamma$. We can easily define coordinatewise a homeomorphism $f : [-1,1]^\Gamma \longrightarrow [-1,1]^\Gamma$ taking $f(\frac{1}{2^n}e_n) = \frac{1}{2}e_n$ and $f(0) = 0$. The sequences $(\frac{1}{2^n}e_n)$ and its image $(\frac{1}{2}e_n)$ weak* converge to 0, but the first is norm convergent and the other is not, not even the sequence of norms converges to 0. $\square$

Apart from this extreme example, in fact it happens that these properties are sensitive to renormings: spaces like $c_0 \oplus_{\ell_1} c_0(\Gamma)$ and $\ell_p \oplus_{\ell_1} \ell_p(\Gamma)$ fail this automatic norm-continuity property despite the fact that they are isomorphic to $c_0(\Gamma)$ and $\ell_p(\Gamma)$, $1 < p < \infty$.

Proposition 5. *Let S be an infinite dimensional separable Banach space and Y be a nonseparable Banach space. Let $X = S \oplus_{\ell_1} Y$. Then, there exists a weak*-to-weak* homeomorphism $f : B_{X^*} \longrightarrow B_{X^*}$ which is not norm-to-norm continuous.*

Proof: Notice that $B_{X^*} = B_{S^*} \times B_{Y^*}$ and then apply Proposition 1. $\square$

Let us point out where the difficulties appear in proving Theorem 2 for say the space $X = c_0(\Gamma)$. Suppose that $f : B_{X^*} \longrightarrow B_{X^*}$ is a weak* homeomorphism. It is a well known fact that the G_δ points of B_{X^*} are exactly the points of the sphere S_{X^*}, hence $f(S_{X^*}) = S_{X^*}$. We know in addition that at the points of the sphere the norm and weak* topology coincide, so we conclude that f is norm-continuous at all the points of the sphere. And this is all that the usual standard functional-analytic techniques can say to us. In order to get norm-continuity at all points

we shall a need a more powerful tool coming from topology: Shchepin's spectral theory. This will allow us to define a certain notion of convergence of a sequence in a compact space of uncountable weight, that we called fiber-convergence. Of course, fiber convergent sequences are respected by homeomorphisms, though not by general continuous functions. Banach spaces in class $\mathcal{E}$ are those for which the fiber convergent sequences in (B_{X^*}, w^*) are exactly the norm convergent sequences, while the class $\mathcal{E}_0$ consists of those spaces in which the fiber convergent sequences of the dual ball are those weak* convergent sequences whose sequence of norms converges to the norm of the limit.

2. Spectral theory

In this section, we summarize in a self-contained way what we need about spectral theory, in the same way as it is exposed in our joint work with Kalenda [1], which at the same time is a reformulation of the ideas from [2] and [3] in a suitable language. Although this preliminary material appears already in [1] with more details, we found it convenient to reproduce it here.

Let K be a compact space. We denote by $\mathcal{Q}(K)$ the set of all Hausdorff quotient spaces of K, that is the set of all Hausdorff compact spaces of the form K/E endowed with the quotient topology, for E an equivalence relation on K. An element of $\mathcal{Q}(K)$ can be represented either by the equivalence relation E, or by the quotient space $L = K/E$ together with the canonical projection $p_L : K \longrightarrow L$.

On the set $\mathcal{Q}(K)$ there is a natural order relation. In terms of equivalence relations $E \leq E'$ if and only if $E' \subset E$. Equivalently, in terms of the quotient spaces, $L \leq L'$ if and only if there is a continuous surjection $q : L' \longrightarrow L$ such that $qp_{L'} = p_L$. The set $\mathcal{Q}(K)$ endowed with this order relation is a complete lattice, that is, every subset has a least upper bound or supremum: if $\mathcal{F}$ is a family of equivalence relations of $\mathcal{Q}(K)$, its least upper bound is the relation given by xE_0y iff xEy for all $E \in \mathcal{E}$, in other words $E_0 = \sup \mathcal{F} = \bigcap \mathcal{F}$. It is easy to check that E_0 gives a Hausdorff quotient if each element of $\mathcal{F}$ does.

Let $\mathcal{Q}_\omega(K) \subset \mathcal{Q}(K)$ be the family of all quotients of K which have countable weight. Notice that $\sup \mathcal{A} \in \mathcal{Q}_\omega(K)$ for every countable subset $\mathcal{A} \subset \mathcal{Q}_\omega(K)$ and also that $\sup \mathcal{Q}_\omega(K) = K$. A family $\mathcal{S} \subset \mathcal{Q}_\omega(K)$ is called cofinal if for every $L \in \mathcal{Q}_\omega(K)$ there exists $L' \in \mathcal{S}$ such that $L \leq L'$. The family $\mathcal{S}$ is called a σ-semilattice if for every countable subset $\mathcal{A} \subset \mathcal{S}$, the least upper bound of $\mathcal{A}$ belongs to $\mathcal{S}$.

Theorem 6 (A version of Shchepin's spectral theorem)**.** *Let K be a compact space of uncountable weight and let $\mathcal{S}$ and $\mathcal{S}'$ two cofinal σ-semilattices in $\mathcal{Q}_\omega(K)$. Then $\mathcal{S} \cap \mathcal{S}'$ is also a cofinal σ-semilattice in $\mathcal{Q}_\omega(K)$.*

It is not so obvious to check whether a given σ-semilattice is cofinal, so this theorem must be applied together with the following criterion:

Lemma 7. *Let K be a compact space of uncountable weight and $\mathcal{S}$ a σ-semilattice in $\mathcal{Q}_\omega(K)$. Then, $\mathcal{S}$ is cofinal if and only if* $\sup \mathcal{S} = K$.

The importance of this machinery is that it allows one to study a compact space of uncountable weight through the study of a cofinal σ-semilattice of metrizable quotients and, in particular, through the natural projections between elements of the σ-semilattice. In this way, the study of compact spaces of uncountable weight is related to the study of continuous surjections between compact spaces of countable weight. The following language will be useful:

Definition 8. Let K be a compact space of uncountable weight and let $\mathcal{P}$ be a property. We say that the σ-typical surjection of K satisfies property $\mathcal{P}$ if there exists a cofinal σ-semilattice $\mathcal{S} \subset \mathcal{Q}_\omega(K)$ such that for every $L \leq L'$ elements of $\mathcal{S}$, the natural projection $p : L' \longrightarrow L$ satisfies property $\mathcal{P}$.

The spectral theorem has the following consequence: In order to check whether the σ-typical surjection of K has a certain property, it is enough to do it on any given cofinal σ-semilattice, namely:

Theorem 9. *Let K be a compact space of uncountable weight, let $\mathcal{P}$ be a property, and let $\mathcal{S}$ be a fixed cofinal σ-semilattice in $\mathcal{Q}_\omega(K)$. Then the σ-typical surjection of K has property $\mathcal{P}$ if and only if there exists a cofinal σ-semilattice $\mathcal{S}' \subset \mathcal{S}$ such that for every $L \leq L'$ elements of $\mathcal{S}'$, the natural projection $p : L' \longrightarrow L$ satisfies property $\mathcal{P}$.*

When the compact space we are dealing with is the dual unit ball B_{X^*} of a non-separable Banach X in the weak* topology, then a cofinal σ-semilattice in $\mathcal{Q}_\omega(B_{X^*})$ can be obtained from a suitable family of separable subspaces of X.

Proposition 10. *Let $\mathcal{F}$ be a family of separable subspaces of X such that*

(1) $\overline{span}(\bigcup \mathcal{F}) = X$, *and*
(2) *if $\mathcal{F}' \subset \mathcal{F}$ is a countable subfamily, then* $\overline{span}(\bigcup \mathcal{F}') \in \mathcal{F}$.

Then the family $\{B_{Y^*} : Y \in \mathcal{F}\}$ is a cofinal σ-semilattice in $\mathcal{Q}_\omega(B_{X^*})$.

Notice that we view B_{Y^*} as a quotient of B_{X^*} for $Y \subset X$ through the natural restriction map. The proof of the proposition is straightforward. In particular, cofinality follows from the first condition and Lemma 7.

3. Fiber convergence

Definition 11. Let $\pi : K \longrightarrow L$ be a continuous surjection, and let (x_n) be a sequence of elements of L converging to $x \in L$. We say that x_n is π-fiber convergent if for every $y \in \pi^{-1}(x)$ there exist elements $y_n \in \pi^{-1}(x_n)$ such that y_n converges to y.

Definition 12. Let K be a compact space of uncountable weight and let (x_n) be a sequence in K that converges to $x \in K$. We say that the sequence (x_n) is fiber convergent if for the σ-typical surjection $\pi : L \longrightarrow L'$, the image of the sequence in L', $(\pi_{L'}(x_n))_{n<\omega}$, is π-fiber convergent.

Definition 13. A nonseparable Banach space belongs to the class $\mathcal{E}$ if the fiber convergent sequences of (B_{X^*}, w^*) are exactly the norm convergent sequences.

Definition 14. A nonseparable Banach space belongs to the class $\mathcal{E}_0$ if the fiber convergent sequences of (B_{X^*}, w^*) are exactly those sequences (x_n^*) weak* convergent to a point $x^* \in B_{X^*}$ such that $\|x_n^*\| \longrightarrow \|x^*\|$.

Notice that Theorems 2 and 3 are immediate consequence of the definitions, because the notion of a fiber-convergent sequence is an intrinsic topological notion and hence it is preserved under homeomorphisms.

Lemma 15. *Let X and Z be Banach spaces and $1 \leq p \leq \infty$. Set $Y = X \oplus_{\ell_p} Z$ and let $\pi : B_{Y^*} \longrightarrow B_{X^*}$ be the restriction map dual to the natural inclusion $X \subset Y$.*

- *If $p = 1$ then every weak* convergent sequence in B_{X^*} is π-fiber convergent.*
- *Suppose that $1 < p \leq \infty$ and (x_n^*) is a sequence in B_{X^*} that weak* converges to x_0^*. Then (x_n^*) is π-fiber convergent if and only if the sequence of norms $(\|x_n^*\|)$ converges to $\|x_0^*\|$.*

Proof: Let (x_n^*) be a sequence in B_{X^*} that weak* converges to x_0^*, and let $y_0^* = x_0^* + z_0^* \in \pi^{-1}(x_0^*)$. If $p = 1$, then $\|x^* + z^*\| = \max(\|x^*\|, \|z^*\|)$ for every $x^* \in X^*$ and $z^* \in Z^*$, hence it is enough to take $y_n^* = x_n^* + z_0^*$ to realize that (x_n^*) is actually π-fiber convergent. If $1 < p \leq \infty$, then norms in the dual are computed as

$$\|x^* + z^*\| = (\|x^*\|^q + \|z^*\|^q)^{\frac{1}{q}}, \quad p^{-1} + q^{-1} = 1$$

Suppose that the sequence of norms $(\|x_n^*\|)$ converges to $\|x_0^*\|$ and let $y_0^* = x_0^* + z_0^*$ be an arbitrary element of the fiber of x_0^*. We will find elements $y_n^* \in \pi^{-1}(x_n^*)$ such that $y_n^* \longrightarrow y_0^*$. If $z_0^* = 0$, then we can simply take $y_n^* = x_n^*$. Thus, we suppose that $z_0^* \neq 0$ and we define

$$\lambda_n = \max\{\lambda \in [0,1] : \|x_n^* + \lambda z_0^*\| \leq 1\}$$

and $y_n^* = x_n^* + \lambda_n z_0^*$. We have to check that $\lambda_n \longrightarrow 1$. Suppose on the contrary that for some subsequence and some $\mu < 1$ we have $\lambda_{n_k} < \mu$. Then

$$(\|x_{n_k}^*\|^q + \mu^q \|z_0^*\|^q)^{\frac{1}{q}} = \|x_{n_k}^* + \mu z_0^*\| > 1$$

so passing to the limit

$$\|x_0^* + z_0^*\| = (\|x_0^*\|^q + \|z_0^*\|^q)^{\frac{1}{q}} > (\|x_0^*\|^q + \mu^q \|z_0^*\|^q)^{\frac{1}{q}} \geq 1$$

which is a contradiction.

Conversely, assume now that the sequence of norms $\|x_n^*\|$ does not converge to $\|x_0^*\|$. Passing to a subsequence, we can suppose without loss of generality that there is a number μ such that $\|x_0^*\| < \mu \leq \|x_n^*\|$ for every n. Let $\xi \in [0,1]$ be such that $\|x_0^*\|^q + \xi^q = 1$, and let z_0^* be any vector of Z^* of norm ξ. We claim that there is no sequence $(x_n^* + z_n^*) \subset B_{Y^*}$ that converges to $x_0^* + z_0^*$. If it were the case, then $z_n^* \longrightarrow z_0^*$, so $\sup\{\|z_n^*\| : n \in \omega\} \geq \|z_0^*\| = \xi$, so

$$\sup\{\|x_n^* + z_n^*\| : n \in \omega\} \geq (\mu^q + \xi^q)^{\frac{1}{q}} > (\|x_0^*\|^q + \xi^q)^{\frac{1}{q}} = 1$$

which contradicts that $x_n^* + z_n^* \in B_{Y^*}$ for every n. $\square$

Theorem 16. *Let $\{X_\alpha : \alpha \in A\}$ be an uncountable family of separable Banach spaces.*

(1) *The c_0-sum $\bigoplus_{c_0}\{X_\alpha : \alpha \in A\}$ belongs to $\mathcal{E}_0$, and if $1 < p < \infty$ then also $\bigoplus_{\ell_p}\{X_\alpha : \alpha \in A\}$ belongs to $\mathcal{E}_0$.*

(2) *All the weak* convergent sequences of the dual ball of $\bigoplus_{\ell_1}\{X_\alpha : \alpha \in A\}$ are fiber convergent.*

Proof: Let $X_0 = \bigoplus_{\alpha \in A} X_\alpha$, where the type of direct sum is the suitable one in each case. Consider I the family of all countable subsets of A and set $X_i = \bigoplus_{\alpha \in i} X_\alpha$ for $i \in I$, and $\mathcal{F} = \{X_i\}_{i \in I}$. This family satisfies conditions (1) and (2) in Proposition 10, and therefore $\mathcal{S} = \{B_{Y^*} : Y \in \mathcal{F}\}$ is a cofinal σ-semilattice in $\mathcal{Q}_\omega(B_{X_0^*})$. We notice that all the natural projections between elements of $\mathcal{S}$ correspond to the dual restriction map of an inclusion of Banach spaces of type $X \subset X \oplus_{\ell_p} Z$, so that Lemma 15 indicates which are exactly the π-fiber convergent sequences in all those cases. Let us focus on part (1). Let $(x_n^*) \subset B_{X^*}$ be a sequence weak* convergent to x_0^*. For $i \in I$ we denote by $\pi_i : B_{X_0^*} \longrightarrow B_{X_i^*}$ the natural surjection dual to the inclusion $X_i \subset X_0$. Let k be a countable subset of A such that $\|x_n^*\| = \|\pi_k(x_n^*)\|$ for all n. We have then that $\|x_n^*\| \longrightarrow \|x_0^*\|$ if and only if $\|\pi_j(x_n^*)\| \longrightarrow \|\pi_j(x_0^*)\|$ for all $j \supset k$, and by Lemma 15 if and only if (x_n^*) is fiber convergent. $\square$

Let KK^* denote the class of Banach spaces with the dual Kadec-Klee property. Then, notice that $\mathcal{E}_0 \cap \mathcal{E} = \mathcal{E}_0 \cap KK^*$. Since $c_0(\Gamma)$ and $\ell_p(\Gamma)$ $(1 < p < \infty)$ have KK^*, we got that these spaces belong to $\mathcal{E}$ and satisfy Theorem 2.

Lemma 17. *Let $K = \prod K_i$ be a finite or countable product of compact spaces of uncountable weight and (x_n) a convergent sequence in K. Then, this sequence is fiber convergent in K if and only if each of the coordinate sequences $(x_n(i))_{n<\omega}$ is fiber convergent in K_i.*

Proof: First of all it is straightforward to check that if $\{\pi_i : X_i \longrightarrow Y_i\}$ is any family of continuous surjections, then a sequence $(y_n) \subset \prod Y_i$ is $\prod \pi_i$-fiber convergent if and only if each coordinate sequence is π_i-fiber convergent. Now, going back to the statement of the lemma, suppose that every coordinate sequence $(x_n(i))_{n<\omega}$ is fiber convergent. Let $\mathcal{S}_i$ be a cofinal σ-semilattice in $\mathcal{Q}_\omega(K_i)$ such that in all surjections π inside $\mathcal{S}_i$, the projection of the sequence $(x_n(i))$ is π-fiber convergent. Let $\mathcal{S}$ be the cofinal σ-semilattice in $\mathcal{Q}_\omega(K)$ formed by all quotients of the form

$\prod \pi_i : \prod K_i \longrightarrow \prod L_i$ where $L_i \in \mathcal{S}_i$. Then, in all surjections π inside $\mathcal{S}$ the projection of the sequence (x_n) is π-fiber convergent. Conversely, suppose that (x_n) is fiber convergent and consider $\mathcal{S}$ the cofinal σ-semilattice in $\mathcal{Q}_\omega(K)$ formed by all quotients which are products of quotients in each coordinate. There exists a cofinal σ-semilattice $\mathcal{T} \subset \mathcal{S}$ such that in all surjections π inside $\mathcal{T}$ the projection of the sequence (x_n) is π-fiber convergent. For every i, we consider $\mathcal{T}_i$ to be the set of all quotients L of K_i such that there is some quotient $\prod_j L_j$ with $L_i = L$. Then $\mathcal{T}_i$ is a cofinal σ-semilattice of $\mathcal{Q}_\omega(K_i)$ and, by the observation at the beginning of this proof, in all surjections π inside $\mathcal{T}_i$ the projection of $(x_n(i))$ is π-fiber convergent. $\square$

Proposition 18. *Let $\{X_n : n < \omega\}$ be a countably infinite family of nonseparable Banach spaces. Then $\bigoplus_{\ell_1}\{X_n : n < \omega\}$ belongs neither to $\mathcal{E}_0$ nor to $\mathcal{E}$.*

Proof: Notice that $B_{X^*} = \prod_{n<\omega} B_{X_n^*}$. For x_n an element of the sphere of X_n^*, the sequence $(x_0, 0, 0, \ldots)$, $(0, x_1, 0, \ldots)$, $\ldots$ is fiber convergent to 0 but not norm convergent. $\square$

Proposition 19. *If $X, Y \in \mathcal{E}$, then $X \oplus_{\ell_1} Y \in \mathcal{E}$.*

Proof: Let $Z = X \oplus_{\ell_1} Y$. Notice that $B_{Z^*} = B_{X^*} \times B_{Y^*}$. A sequence in this product is fiber-convergent if and only if both coordinates are fiber-convergent and the same happens for norm-convergence. $\square$

We can provide a couple of extra examples. In both cases $1 < p < \infty$ and c_0 can be substituted for ℓ_p:

- $\ell_p(\Gamma) \oplus_{\ell_1} \ell_p(\Gamma) \in \mathcal{E} \setminus \mathcal{E}_0$. This space belongs to $\mathcal{E}$ by Proposition 19, but it is not in $\mathcal{E}_0$ because it is in $\mathcal{E}$ but it fails the dual Kadec-Klee property: If (x_n^*) is a sequence inside the unit sphere of $\ell_p(\Gamma)^*$ which weak* converges to 0, then the sequence (x_0^*, x_n^*) shows that KK^* does not hold.
- An uncountable ℓ_p-sum of copies of $\ell_p(\omega) \oplus_{\ell_1} \ell_p(\omega)$ belongs to $\mathcal{E}_0 \setminus \mathcal{E}$. This space belongs to $\mathcal{E}_0$ by Theorem 16 but again it is not in $\mathcal{E}$ since it is in $\mathcal{E}_0$ but it fails KK^* for similar reasons as in the previous case.

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